5. This geometrical algorithm may easily be translated into an algebraical algorithm and thus furnish an elementary arithmetical method of extracting roots by a method of very rapid convergence. The general algorithm thus obtained is as follows:

$$\frac{x_1}{y_1}=\frac{(n-1)x^n+y^n z+nzxy^{n-1}}{nx^{n-1}y+x^n+(n-1)y^n z}, \qquad (1)$$

where n denotes degree of root, z the given number, the root of which is to be found, $x \div y$ an arbitrary initial value and $x_1 \div y_1$ the succeeding corrected value.

By dividing the right side of (1) by y^n and substituting a for $x \div y$, and a_1 for $x_1 \div y_1$ we obtain another form of the same algorithm:

$$a_1=\frac{(n-1)a^n+z+naz}{na^{n-1}+a^n+(n-1)z} \qquad (2.)$$

In regard to this algorithm the following observation may be allowed. There are the two following algorithms of a more simple form, i. e.

$$a_1=\frac{(n-1)a^n+z}{na^{n-1}} \quad (3,) \text{ and } \quad a_1=\frac{naz}{a^n+(n-1)z} \quad (4.)$$

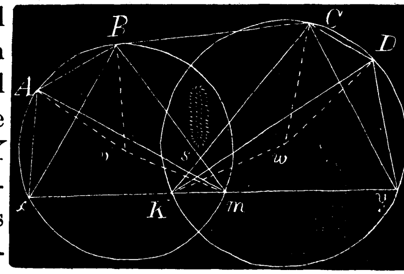
If a be again an arbitrary approximate value for the n^{th} root of z, then the corrected value a_1 in (3) always furnishes a value greater than the n^{th} root of z; and the value in (4) always is smaller than the n^{th} root of z. The acceleration of both methods is of the same (quadratic) order. Now by addng both numerators and dividing by the sum of both denominators we obtain a mean value between the two in (3) and (4); that is, another approximate value for the n^{th} root of z of at least the same degree of approximation. But this result exactly coincides with the algorithm under (2), derived from geometrical constructions.

SOLUTION OF A PROBLEM IN SURVEYING.

BY T. J. LOWRY, U. S. C. S. SAN FRANCISCO, CAL.

Problem:—Four points in the same plane being given in position to determine the position of any other two intervisible points (or places of observation) in reference to these points, having from each place of observation the angles included between the other place of observation and each of two of the known points, and with the known points so situated that the two which are visible from the first point of observation are not visible from the second, and *vice versa*.

Let A, B, C and D be the four given points, and AxB, Bxy Cyx and CyD be the observed angles.



In the $\triangle ABm$ are given AB, and the $\angle\ ABm (=$ supplement of the sum of the observed $\angle$s AxB and Bxy), and the $\angle BmA (=$ observed $\angle AxB$), we can hence find Bm. And in $\triangle DCK$ are known CD, and the $\angle DCK$ (= supplement of the sum of the observed $\angle$s DyC and Cyx), and the $\angle\ CKD$ (= observed $\angle\ CyD$) to find CK.

Now the $\angle SBC = \angle ABC - < ABm$, and $< SCB = < BCD - < DCK$ $\therefore$ in $\triangle BSC$ we have BC and all the angles to find BS and SC. But $mS = Bm - BS$, and $KS = KC - SC$ and the $< KSm = < BSC$ $\therefore$ in $\triangle\ SKm$ are known two sides SK and mS and the included angle, to find angles SmK and SKm. Now in $\triangle\ Bmx$ are given Bm, and the $<$ s Bmx and Bxm to get Bx. And then in $\triangle\ ABx$ we have Bx, and AB and the $< AxB$ to determine Ax. Also in $\triangle\ KCy$ are given KC, and $<$ s CKy and CyK to find Cy. Then in $\triangle\ CyD$ are known Cy, CD and the $< CyD$ to get Dy.

Since the $<$ s wDC and wCD are each equal to the compliment of the observed $< DyC$ the rule for laying down the circle of positon through C and D is obviously to lay off from CD at the points C and D the compliment of the observed $< CyD$ and the point of intersection of the produced sides of these $<$ s will be the center of a circle of position. Now about this point with radius Cw (or Dw) sweep the circle of position; and in like manner lay down the other circle of position through A and B. Then lay off from Cw at the point w twice the observed $< Cyx$ and produce the side wK till it intersects the first circle of position in some point as K (which will be a point in the line of sight through x and y): also from Bv at the point v lay off twice the observed angle Bxy and the point m where the side vm produced intersects the second circle of position will be another point in the line joining x and y. Now through K and m draw a right line and produce it each way until it intersects the circle of position and the points of intersection x and y will be the required places of observation.

But with the aid of the "three arm Protractor" this problem can be plotted much more expeditiously (and without laying down circles of position) as follows :—with the $<$ s ABx, and Bxy set off respectively on the left and right limbs of the protractor, cause the fiducial edges of the left and middle arms to traverse A, and B, and draw a line along the true edge of its right

arm, then shift the center of the protractor (taking care to keep the true edge of the left and middle arms bisecting the points A and B) and draw another line along the true edge of the right arm and the point of intersection of the two lines thus drawn will be a point in the line of sight joining x and y. Now with the $<$s DyC and Cyx set off on the right and left limbs of the protractor, shift its center till the true edges of the right, middle and left arms traverse D, C and K, dot the center and we have y (one of the places of observation). And again with the $<$s AxB and Bxy on the left and right limbs, place the true edge of the right hand arm on the line Kmy and shift the center along this line till A and B are traversed by the true edges of left and middle arms then dot the center and you have x (the other place of observation).

The Hydrographer, the Topographer and the Explorer will each find this problem servicable.

ODD NUMBERS AND EVEN NUMBERS.

BY ARTEMAS MARTIN, ERIE, PA.

All numbers are either odd or even. An *even* number is a number that can be divided by 2 without a remainder; an *odd* number is one that is not divisible by 2. 1, 3, 5, 7, are *odd* numbers; 2, 4, 6, 8, are *even* numbers.

All even numbers are comprised in the formula $2n$, and all odd numbers in either of the formulæ $2n + 1$, $2n - 1$.

Proposition I. — The sum of two even numbers is even.

Proof. — Let $2m$ and $2n$ represent any two even numbers; their sum is $2m + 2n = 2(m + n)$, which is even.

Prop. II. — The sum of two odd numbers is even.

Proof. — Let $2m + 1$ and $2n + 1$ be any two odd numbers; their sum is $2m + 1 + 2n + 1 = 2m + 2n + 2 = 2(m + n + 1)$.

Prop. III. — The sum of an odd number and an even number is odd.

Proof. — Let $2m + 1$ be any odd number and $2n$ any even number; then $2m + 1 + 2n = 2(m + n) + 1$, which is odd.

Prop. IV. — The difference of two even numbers is even.

Proof. — $2m - 2n = 2(m - n)$.

Prop. V. — The difference of two odd numbers is even.

Proof. — $2m + 1 - 2n - 1 = 2(m - n)$.

Prop. VI. — The difference of an odd number and an even one is odd.

Proof. — $2m + 1 - 2n = 2(m - n) + 1$.